SPECIAL THEORY OF RELATIVITY AND RELATIVISTIC THERMODYNAMICS

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Abstract: Galilean transformation equations are proven wrong by the special theory of relativity discovered by A. Einstein. Two inertial observer moving with respect to each other measure quantities such as length, time, mass, kinetic energy to be different. Maxwell-Boltzmann distribution changes under relativistic condition to Maxwell-Juttner distribution under relativistic conditions. But transformation of temperature at high speeds is not agreed upon. The study of transformation of temperature based on kinetic theory of ideal gas is presented.

1. Introduction

According to Galilean Transformation, the speed of light is different in two inertial frames, moving with constant velocity with respect to each other. In the late 18th and early 19th century, there was considerable evidence supporting the idea that the speed of light is an invariant quantity with respect to all inertial observers. In 1905, Einstein made the ground breaking discovery that when transforming physical quantities from one inertial observer's frame to another, the Lorentz transformations must be used so that the speed of light remains an invariant quantity. The principle of relativity modified the Newton's laws of motion and in a sense unified concepts like space and time. Since then, much effort has been devoted to the search for the relativistic forms of other laws of physics. While transforming temperature, it can be seen that the results in the current literature, have been obtained by postulating the covariance of the first and/or second law(s) of thermodynamics under Lorentz transformation.

2. Postulates of Special Theory Of Relativity

In 1905, Einstein published a paper on the relativity, in which he dismissed the problem of absolute motion by denying its existence. He proposed two postulates.

1. The laws of physics are the same in all inertial frames.
2. The speed of the light in free space has same value 'c' in all inertial frames of reference.

The consequences of this theory are length contraction, time dilation, breaking of simultaneity.

The laws of motion are reconstructed to be coherent with the laws of electrodynamics.

These new equations of motion become Newtonian equations when $v/c \ll 1$

If co-ordinates of space-time in S frame are x, y, z, t and co-ordinates of space-time in S' frame are x', y', z', t' and if S' moves with respect to S along x axis with velocity v,

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$

$$z' = z$$

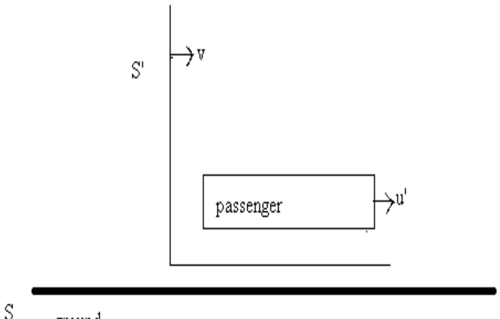
$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

2.1 Length Contraction

A body's length is measured to be greatest when the body is at rest with respect to the observer. When it moves with velocity v relative to the observer, its measured length is contracted in the direction of motion by the factor of $(1-v^2/c^2)^{1/2}$.

2.2 Time Dilation

A body's clock runs fastest when measured by an observer at rest with respect to body.
Relativistic Addition of Velocities



Relativistic addition of velocities

Let S be the ground frame which at rest and S' be the frame of the train, whose speed relative to ground is v. The passenger speed in train is u'.If we call passengers speed relative to ground u, then his ground location as time goes on is given by,

$$u = \frac{(u' + v)}{1 + \frac{u' v}{c^2}}$$

2.3 Relativistic Mass

A body's mass is measured to be greatest when the body is at rest with respect to the observer.

If an object with mass(rest massm_0)is moving with velocity u with respect an observer, then its mass measured by the observer is given by,

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

2.4 Relativistic Energy

When an object is at rest, it possesses some energy called as rest energy which is equal to m_0c^2. It is the energy possessed by an object by the virtue of its mass. Photon which has no mass has zero rest energy. Rest energy is different from internal energy. Unlike internal energy, rest energy can't be added or removed from the object. Internal energy of an object depends on its temperature, whereas rest energy is same at all temperatures (even at zero Kelvin!).

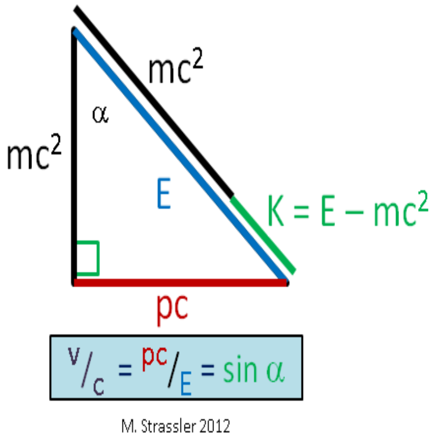
Total energy of a particle (E) is given by,

$E = m_0c^2 + K.E$

This equation can be rewritten as

$E^2 = (mc^2)^2 + (pc)^2$

Above equation is famously represented as a Pythagorean triangle.



3. Maxwell–Jüttner distribution

The Maxwell–Boltzmann distribution or describes particle speeds in gases, where the particles move freely without interacting with each other.

It is a probability distribution (derived assuming isotropy) for the speed of particles constituting the gas. Particles move freely between short collisions and do not interact with each other. The particle can have any speed ranging from zero to infinite, but is more likely to be within one range of some speeds than others. The speed at which particle picked randomly most likely to be travelling is called as most probable speed. Most probable speed is a function of temperature. As the temperature increases, most probable speed also increases. But this distribution is applicable only at non relativistic speeds (v<<c).

Maxwell–Boltzmann distribution is generalized at relativistic speeds (speeds comparable to the speed of light) to Maxwell–Jüttner distribution. The Maxwell–Jüttner distribution considers a classical ideal gas where the particles do not significantly interact with each other. Effects of special relativity are taken into account. In the limit of low temperatures $T << mc^2/k$ (where m is the mass of particle making up the gas,
k is Boltzmann's constant and c is the speed of light), this distribution becomes identical to the Maxwell–Boltzmann distribution.
As the gas becomes hotter and kT approaches or exceeds mc^2, the probability distribution for

$$\gamma = 1/\sqrt{1-\frac{v^2}{c^2}}$$

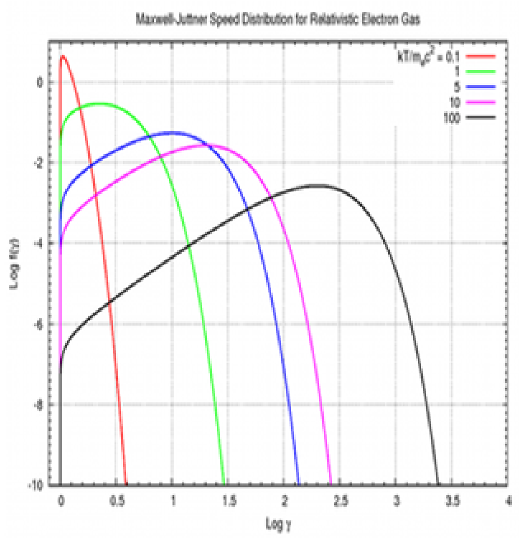
(γ being Lorentz factor) is given by the Maxwell–Jüttner distribution:

$$f(\gamma) = \frac{\gamma\beta e^{-\frac{\gamma}{\theta}}}{\theta K_2\left(\frac{1}{\theta}\right)}$$

Where,

$$\beta = \frac{v}{c},\quad \theta = \frac{kT}{mc^2}$$

and k_2 is the modified bassel function of the second kind.



Maxwell–Jüttner distribution over Lorentz factor (relativistic Maxwellian), for a gas at different temperatures. Speed is represented in terms of the Lorentz factor.
As The value of kT/mc^2 approaches 0, the distribution becomes maxwell boltzmann distribution
As The value of kT/mc^2 increases, the distribution deviates from maxwell boltzmann distribution.

4. Relativistic Thermodynamics

Following Einstein's relativity theory in 1905, all contemporary physics was rapidly 'relativized'.

There are three possible effects of Special Theory of Relativity (STR) on a thermodynamic system.(Popovic et al) Einstein and Planck looked upon this process as isobaric; on the other hand Ott saw it as an adiabatic process (Ott et al) and Landburg saw the process to be isothermal.

Thermodynamics was adapted to the new covariance requirements by Planck and Einstein, and there the matter rested, appearing to be complete and satisfactory. They applied principle of the least action to a moving black-body cavity and assumed the first and the second laws of thermodynamics (1), (2) to be covariant:

(Symbols T, P, Q, S, W and U are used for temperature, pressure, heat, entropy, work and internal energy, respectively.)

$dU = dQ + dW$ (1)

$dQ = T\,dS$, (2)

Besides, they proposed the following set of transformations:

$T = T'/\gamma, Q = Q'/\gamma$ (3)

$S = S', P = P'$ (4)

In which primed quantities represent, the ones measured in the proper frame of reference that is moving with constant velocity $\vec{v}$ with respect to a stationary reference frame and γ is:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

In 1963 Ott dug out the problem and claimed to demonstrate the inverse of Planck's transformation laws, arguing that a moving observer would judge the system to be hotter and heat flux to be larger:

$T = \gamma T'$, $Q = \gamma Q'$ (6)

$S = S'$, $P = P'$ (7)

Two years later Arzelies (Arzelis et al) independently concluded the same. Landsburg, however, then dissented from the new orthodoxy, claiming that temperature is a Lorentz invariant whereas Planck's original transformation for the heat remains correct:

$T = T'$, $\gamma Q = T\,dS$ (8)

$Q = Q'/\gamma$ (9)

$P = P'$ (10)

We can summarize above discussion as the following:

$T = \gamma^{\alpha} T'$ (11)

Where $\alpha = -1, 0, 1$ comes from Planck/Einstein, Landsberg and Ott views, respectively.

One way to approach this problem is by considering kinetic theory of ideal gases. (Khalegy et al) By Boltzmann probability distribution for a classical ideal gas in a thermal bath, average energy is related to the temperature of reservoir, T, as below:

$$E_{avg} = \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E e^{-\frac{\beta E d^3 x d^3 p}{h^3}}}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{\beta E d^3 x d^3 p}{h^3}}} \quad (12)$$

Where h is a normalization constant with dimensions of action ,$\beta = 1/(kT)$ and k is the Boltzmann constant. According to the non-relativistic expression for kinetic energy, $p^2/2m$, E_{avg}= (3/2)kT, where the upper limit for the integral over momentum goes to infinity which requires us to use the relativistic form of energy. Relativistic form of energy has to be used in astrophysics and plasma physics. E.g. When the kinetic energy of high energy electrons is measured using non-relativistic equations, electron energies show 5%-15% less value than the actual value. (Kasthurirangan et al.)

Energy-momentum relationship in relativistic form

$E^2 = (mc^2)^2 + (pc)^2$ (13)

Using Bessel's function and equation (13) variables are changed

$p = mc \sinh(\chi)$ (14)

and E_{avg} is calculated as

$$E_{avg} = mc^2\left[\frac{k_1(u)}{k_2(u)} + \frac{3}{u}\right] \quad (15)$$

k_1 and $k_2$are modified Bessel functions and u = βmc^2

$$k_1(u) \cong \left(\frac{\pi}{2u}\right)^{\frac{1}{2}} e^{-u}\left[1+\frac{3}{8u}\right] \quad (16)$$

$$k_2(u) \cong \left(\frac{\pi}{2u}\right)^{\frac{1}{2}} e^{-u}\left[1+\frac{15}{8u}\right] \quad (17)$$

By using (15), (16), (17)

$E_{avg} - mc^2 = E_k = (3/2)kT$

Where E_k is kinetic energy.

For obtaining the relativistic transformation of temperature, average energy in any frame of reference is related to its temperature by Boltzmann probability distribution function is presumed.

Let some arbitrary thermodynamic system is placed in S' frame moving with velocity v relative to lrest frame. T' is the temperature of the system measured in the S' frame. What is the temperature T as measured in the rest frame compare to T'? From 4-vector momentum transformation we have

$$E_{avg} = \gamma\left(E_{avg}{}' + v.p_{avg}\right)$$

$p_{avg} = 0$ in S' frame. (motion is random in S' frame)

Substituting average energies by their related temperatures

$$\frac{k_1(u)}{k_2(u)} + \frac{3}{u} = \gamma\left[\frac{k'_1(u)}{k'_2(u)} + \frac{3}{u'}\right]$$

Where u = mc^2/kT and $u' = mc^2/kT'$

In both classical (u ≫1) and relativistic (u ≪1) order above equation, reduces to Ott transformation:

$\gamma \to 1 \Longrightarrow T = T'$

$\gamma \to \infty \Longrightarrow T = \infty$

6. Conclusion

The transformation derived here predicts that moving objects appear hotter to stationary observers. This is in accord with Ott's view. At the present time no temperature transformation has been agreed upon. To reach unanimity, firm experimental evidence is needed.

7. References

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